This is the Final Manuscript version of an entry titled "Relative numerousness" to appear in the forthcoming *Encyclopedia of Animal Cognition and Behavior.* The entry was invited, it has been accepted, and the typeset version that will be available electronically and in print has been provided to the author.

It seems unlikely that the *Encyclopedia* will be published before 2020 or possibly later, so final page numbers will not be known to this author until the electronic and print versions of the entire *Encyclopedia* are available.

March 23, 2019   Roger K, Thomas, PhD

Title of Entry: Relative numerousness

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Introduction

Stevens (1951, p. 22) distinguished between numerosity and numerousness. Numerosity referred to the determination of the cardinal number of a set of objects by counting, and numerousness referred to the determination of the cardinal number for a set of objects by a method other than counting. Stevens' distinction is used here, although many investigators do not, and they often use numerosity in instances where numerousness should be used.

What is Counting?

Counting research involving children and nonhuman animals usually relies on whether the child or animal has met the requisite number of Gelman and Gallistel's (1978) five principles of counting. In order as listed, the five principles are:

1. Stable Order – say you have five apples, you must always count them in the same order (e.g., 1-2-3-4-5).

2. One-to One Correspondence – each object receives one and only one count. Gelman and Gallistel also wrote that "tagging" each object with a unique tag was a part of this principle (again, e.g., "1," "2," "3," "4," "5").

3. Cardinality – the last tag correctly applied represents the quantity for that set of objects.

4. Abstraction – the ability to count any set of objects, e.g., apples, oranges, cows. etc.

5. Order Irrelevance – using the example of five apples side by side. You may start at either end, in the middle, etc. but you must count each apple only once and end with the correct cardinal number of apples.

Most investigators, including Gelman and Gallistel, consider that if the first three principles are met, one has provided evidence that the child or animal can count.

In most animal research, it is questionable whether an animal has learned the necessary tags to be able to count. Perhaps, research from the laboratories of Sarah Boysen, Duane Rumbaugh, and Tetsuro Matsuzawa have come the closest; all studied chimpanzees. Most animal research associated with animals' use of number has not involved counting as no tagging system is used. An alternative process to counting that most animals likely use will be described below.

Interest in animals' use of number achieved prominence in the early 20th century with well-known research on "Clever Hans," the counting horse. Eventually, it was determined that Hans's apparently successful performances were based on inadvertent

cues by his owner, and that gave birth to the need to prevent "Clever Hans cues" in animal learning and cognition research.

Ecological Salience of Numerousness

James Lipton (1991) is a modern authority on nouns of assembly (e.g., a gaggle of geese, a colony of ants, a flock of sheep). Lipton reported that the earliest known list was *The Edgerton Manuscript* (circa 1450), and the best known of the early compilations was the *Book of Saint Albans* (1486). Most nouns of assembly were associated with game and domestic animals, but humans as well (e.g., a den of thieves). The history of nouns of assembly shows the long-time interest in the uncounted numerousness of animals. In some cases, a noun of assembly was shared by several species, such as, "herd" or "flock."

Numerousness has ecological salience for many animals. For example, the monkey in a group of monkeys feeding competitively that first detects several bushes of edible fruit is likely to go first to the bush perceived to hold the most fruit and, with clusters of fruit on a single bush, is likely to go for the cluster with the most fruit. A predator, such as a lion, that feeds on prey that depend on escape for defense such as wildebeests, will be more successful by attacking a large group, as it will be more likely to yield a young, weak, or lame wildebeest. Leopards that prey upon baboons will seek a smaller group to attack, as baboons in significant numbers are more likely to fend off the predator. The numbers of eggs and hatchlings for most nesting birds or the number of offspring for other animals giving multiparous birth likely find numerousness salient. Imagine the mother with multiple altricial offspring trying to keep an account of the proximity of her young. Of course, it seems unlikely that any of these animals meet

Gelman and Gallistel's prerequisites for counting, especially that of having a "tagging system." So what might they use to process number?

Processes Used to Determine Numerousness

Davis and Perussé (1988) presented an ambitious glossary of numerical processes. In the following order, Davis and Perussé's glossary included (a) Relative numerousness judgments, (b) Subitizing, (c) Estimation, and (d) Counting, which they further divided into (e) Protocounting, and (f) Concept of number.

Thomas and Lorden (1993) countered that only three types of numerical processes were necessary: (a) absolute numerousness judgments, (b) relative numerousness judgments, and (c) counting. See Thomas and Lorden (1993) for reasons why they disagreed with Davis and Perussé's glossary.

Absolute numerous judgments by squirrel monkeys were investigated by Thomas, Fowlkes, and Vickey (1980). They used cards with black-filled circles (hereafter "dots") and all the appropriate controls. After lengthy, intermingled stepwise training and testing, their two squirrel monkeys learned to discriminate seven from eight dots; one monkey discriminated eight from nine dots. Thus, both monkeys learned that "sevenness" was different from "eightness." The monkeys could not count the dots as they had no opportunity to learn a "tagging system" (see "What is Counting?" above).

With practice, humans may become adept at accurately and immediately perceiving as many as seven or eight dots. Thomas, Phillips, and Young (1999) showed that most humans tested could identify seveness and eightness under conditions that prevented counting. A few were accurate with higher numbers. The upper limit may be related to G. A. Miller's well-known "magical number seven, plus or

minus two," which Miller said specified the limits of unidimensional information processing; see discussion in Thomas et al. (1999).

Children and adults, especially in Western cultures, have considerable experience recognizing numerousness accurately without counting. Consider the number of dots on the six faces of dice used in many board games. Most will recognize the number of dots (I – 6) on the top face without counting, and if two dice are used, most will immediately perceive the sum of the dots on the two top faces. This ability can be acquired with less fixed patterns. For example, if you approach a bus stop with three people waiting, you will likely perceive "three" without counting them. If a group of four birds flew overhead, you will likely perceive "four" without counting. If you saw five sheep grazing, in a cluster, you will likely perceive "five" without counting, and when in doubt, one may easily count them. Go out and see for yourself whether you can immediately perceive low numbers of objects, animals, people in clusters, etc. without counting and whether with practice, as in the Thomas et al. (1999) study, you can learn to perceive as many as eight objects without counting.

Making relative numerous judgments without counting is usually easier if you do not need to know the exact numbers. For example, if driving along a country road, you see a small pasture on the left with 20 cows and, on the right, you see a similarly small pasture with 50 cows, you likely perceive immediately that there are more cows on the right. Smaller differences might be perceived immediately, say 20 versus 30 cows, but 20 versus 23 cows might require counting. Later, an experiment that investigated relative numerous judgments by squirrel monkeys will be described.

Prototype Matching

Inspired by Rosch (1975), Thomas and Lorden (1993) proposed that accurate perception of numerousness is done by prototype matching. Among other semantic categories, Rosch used 53 species of birds (and the mammal, bat; see page 232) as examples of the category "Bird." For her American students, the best prototype of a bird was the American robin; the penguin and the bat were deemed to be farthest from the prototypical robin. Based on research by Thomas et al. (1980), Thomas and Lorden concluded that squirrel monkeys acquired prototypes for as many as eight discriminanda. Thomas et al. (1999) extended that conclusion to humans.

Conceptual Relative Numerousness Judgments

Apparently. the first published study using animals (three squirrel monkeys), to demonstrate the ability to make conceptual relative numerousness judgments was by Thomas and Chase (1980). Like Thomas et al. (1980) they also used "dot" cards as discriminanda, and they used all the appropriate controls. Three cards were presented on a platform. The monkey responded by pushing aside a card holder that exposed a food well. If it was correct, a food reinforcer (a currant) was easily accessible from the food well. On the front of the platform was a row of three 25-watt, neon panel lamps. If all lamps were illuminated, the correct response was to choose the card with the highest number of dots. If the two end lamps were illuminated, the correct response was to choose the card with the intermediate number of dots, and if only the single center lamp was illuminated, the correct response was to choose the card with the fewest dots. Position (left, center or right) of the card bearing the correct numerousness card was randomized.

Sixteen intermingled training and testing steps were used. In order, the most definitive steps were to learn to (a) choose the highest number when three lamps were on, (b) choose the intermediate number when two lamps were on, (c) choose between random presentations of highest and intermediate numbers according to the cue lights, (d) choose the lowest number when one lamp was on and (f) choose among the highest, intermediate and lowest dot-number cards presented in random order and consistent with the cue lights.   One monkey succeeded on the final step; that is, it could choose correctly the card with the highest, intermediate, or lowest number of dots as cued by the lights. Another monkey was able to choose correctly between highest and intermediate numbers of dots presented randomly, but the third monkey learned only to choose the most dots. Nevertheless, all monkeys showed some ability to respond to relative numerousness conceptually, and the best monkey's performance indicated that it made ordinal judgments involving relative numerousness.

Concluding Remarks

Animals including humans accurately identify as many as eight discriminanda without counting. They do this by acquiring prototypes ("fourness," "eightness," etc.) and applying them accurately. Relative numerousness is judged easily when two or more sets of discriminanda are sufficiently different in number and are distinctly clustered.

Because at least two squirrel monkeys in Thomas and Chase's (1980) study learned to choose the intermediate-in-numerousness set of discriminanda when three sets (most, intermediate, and fewest) were presented concurrently, Thomas and Chase

(1980) also concluded that squirrel monkeys learned "ordinal numerousness judgments," a higher-order example of relative numerousness judgments.

However, subsequent researchers, have not followed Thomas and Chase's condition that ordinal judgments should be based on a minimum of three sets of numerousness discriminanda being presented concurrently, and some investigators refer to their animals having made "ordinal judgments" when only two sets of discriminanda were presented (e.g., Olthof, Iden, & Roberts, 1997, who tested squirrel monkeys; Washburn & Rumbaugh, 1991, who tested rhesus monkeys).

However, Beran, Beran, Harris, & Washburn (2005) conducted a clever experiment testing two chimpanzees and a rhesus monkey. They began their article by quoting Brannon's (2002) definition of "ordinality."

> *Ordinality* refers to the relative position of one entity with respect to other entities in a sequence . . . . an understanding of ordinality implies not only differentiating two sets of objects or symbols from each other but also knowing what set or symbol is numerically larger or smaller (Brannon 2002) (Beran et al. 2005, p. 351).

However, Beran et al. went beyond Brannon's definition and used five differently colored plastic eggs, each covering a different number of identical food items. Initially, only two eggs were presented at a time, but eventually trials were presented with three, four or five eggs, and the primates were able to choose the eggs in the order that covered the most food items, the next-most, etc., which indicates that the primates had learned the optimal order of the egg-color, food-items relationship.

Cross-References

Absolute Number Discrimination

Cardinality

Comparative Cognition

Concept Formation

Number Concept

Numerosity

Numerousness

Predator Defense

Prey

Prototype Theory

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